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II-317 A Dynamical System Model and a Filter Simulation for a Transmission System with Multiplicative and Additive noises

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Abstract

Recently, the interest is increasing to the multiplicative noises in communication and control systems. This paper shows the method of the estimate of a phase modulated random signal in a transmission system with the multiplicative and also additive noise using the state estimate method. The system is assumed to be continuous. A procedure to construct a dynamical system model for such a transmission system is presented, and a filter simulation concerning the estimate of a phase modulated random signal is also performed by assuming the first-order approximation for a nonlinear filter.

〔1〕 Introduction

The noises in a transmission system (a communication system) are usually treated as additive noises which are independent of the transmitted signal as are the thermal noise. Moreover both signal and noise in a transmission system are assumed to be approximate stationary random processes. Then optimal filtering (signal estimates) problem, being very important to transmission systems, is reduced to Wiener filter theory[(1)] which is available for the systems being scalar and stationary.

Recently the interest to the non-additive noise in transmission systems is increasing[(2)(3)]. Such a noise is dependent on transmitted signal and occur in optical communication systems[(2)], fading channel[(4)] and others[(5)(6)]. Then optimal filtering problem for a transmission system with non-additive noise is very interesting and has already been discussed by the author in connection with the filtering to the systems in which the non-additive noise is expressed by the nonlinear coupling noise[(7)(8)]. Now if a system becomes nonstationary random processes or the system is nonlinear, the filtering via Wiener theory requires much effort to obtain the solution. In that case, the filtering via state estimate method is more useful than Wiener theory for obtaining the solution.

In this paper we show an application of the filtering by state estimate theory to the estimate of the phase modulated random signal message in a continuous transmission system with multiplicative and additive neises. Such an application is practically important. The results of this paper are meaningful, since the application of the state estimate method to such a system is only devoted to the estimate of the typical no mudulated gaussian random signal message. Finally we show a numerical experiment of the estimate of such a signal message in the system. The mathematical rigor is omitted in the present discussion which is intended to be sufficient in practice.

〔2〕 Transmission system model with multiplicative noise and additive noise

A noise being dependent on the transmitted signal is usually repersented by an analytic functional F [t,

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r (t)] where r (t) denotes the received or transmitted signal and t is time variable. $F[t, r(t)]$ can be expanded as

$$F[t, r(t)]=\sum_{l} m_l(t) r^l(t) \tag{1}$$

where $m_1(t)$ is generally random processes. Thus we can understand $F[t, r(t)]$ is a noise being dependent on $r(t)$. Now the most simple formula of $F[t, r(t)]$ is a multiplicative formula $m_1(t)\ r(t)$. We call here $m_1(t)$ a multiplicative noise. Moreover a transmission system usually includes additive noise such as thermal noise as well as a multiplicative noise. **Fig. 1** shows a model of such a transmission system which is called a channel model. In the figure $a_1(t)$ is a transmitted scalar signal. The linear filter also shown in the figure

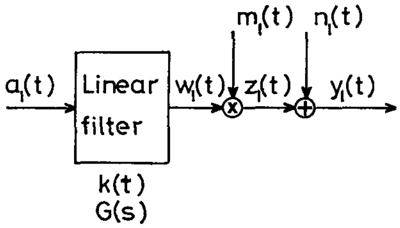

Fig. 1 Transmission system model

is represented either by an impulse response $k(t)$ or its transfer function $G(s)$ by which the characteristics of the transmission line is expressed. The output scalar signal of the filter is denoted by $w_1(t)$. $m_1(t)$ is a multiplicative noise as mentioned above and $z_1(t)$ is equel to $w_1(t)\ m_1(t)$ which is thus the noise being dependent on the signal. $n_1(t)$ is an additive noise and $y_1(t)$ is a received signal which is the final output of the transmission system. Here we must note that $a_1(t)$ means only the first component of vector signal $a(t)$. Therefore, we assume that the same notation is applied to the other input or output components. Then a multiplicative noise which is the most simple non-additive noise is very interesting. The transmission system model shown in **Fig. 1** is a typical model which is practically realizable and applicable for an optical fiber transmission system if some assumptions are made[2][9].

[3] A construction of dynamical system model for a transmission system

Now we construct a dynamical system model without general theory for the transmission system in **Fig. 1**

(a) Transmitted signal

Transmitted signal $a_1(t)$ is assumed to be PM (Phase modulation) wave to a random signal message $\xi_1(t)$, where $\xi_1(t)$ is given by the following Gauss-Markov process or Langevin equation.

$$\dot{\xi}_1(t)=-A\ \xi_1(t)+G_\xi\ u_1^{(\xi)}(t) \tag{2-1}$$

$$\xi_1(t_0)=\xi_{1,0} \tag{2-2}$$

where symbol $\cdot$ means the time derivative, t_0 is initial time, and A, $G_\xi>0$. $\xi_{1,0}$ is the initial value of $\xi_1(t)$. Moreover the input $u_1^{(\xi)}(t)$ is assumed to be a white gaussian noise with zero mean and the covariance $q^{(\xi)}$ which is defined in the following.

$$E[u_1^{(\xi)}(t)\ u_1^{(\xi)}(\tau)]=q^{(\xi)}\delta(t-\tau) \tag{3}$$

where E means a mathematical expectaion and $\delta(t)$ is Dirac delta function. $q^{(\xi)}$ is non-negative constant. The initial value $\xi_{1,0}$ is assumed to be a gaussian random variable with the mean $\overline{\xi_{1,0}}=E[\xi_{1,0}]$ and the non-negative constant variance $p_0^{(\xi)}$ which is given as follows.

$$p_0^{(\xi)}=E[(\xi_{1,0}-\overline{\xi_{1,0}})^2] \tag{4}$$

$\xi_{1,0}$ and $u_1^{(\xi)}(t)$ are assumed to be statistically independent each other. And then the transmitted signal $a_1(t)$ is defined as follows.

$$a_1(t) = C\sin\ (\omega_0\ t + d\xi_1(t)) \tag{5}$$

$$C,\ d > 0,\quad \omega_0 = 2\pi f$$

where f is the carrier frequency.

(b) Linear filter

Now we assume that the linear filter representing the characteristics of the transmission line is a low-pass filter and its impulse response $k(t)$ is given by

$$k(t) = k_1\ \exp\ (-k_2\ t) \tag{6}$$

$$k_1,\ k_2 > 0$$

Then transfer function $G(s)$ corresponding to $k(t)$ is easily obtained as

$$G(S) = \frac{k_1}{s + k_2} \tag{7}$$

where s is a complex number. Therefore we can obtain the following dynamical system from the input-output relation of the linear filter.

$$\dot{\alpha}_1(t) = -k_2\alpha_1(t) + k_1 a_1(t) \tag{8-1}$$

$$\alpha_1(t_0) = \alpha_{1,0} \tag{8-2}$$

$$w_1(t) = \alpha_1(t) \tag{9}$$

and initial value $\alpha_{1,0}$ is assumed to be a gaussian random variable with the mean $\overline{\alpha_{1,0}} = E[\alpha_{1,0}]$ and the nonnegative constant variance $P_0^{(\alpha)}$ such as

$$P_0^{(\alpha)} = E[(\alpha_{1,0} - \overline{\alpha_{1,0}})^2] \tag{10}$$

(c) Multiplicative noise

The determination of the statistical properties of the multiplicative noise $m_1(t)$ is not so easy without the identification of $m_1(t)$. But here we define $m_1(t)$ as follows.

$$m_1(t) = m_1^{(1)}(t) + m_1^{(0)} \tag{11}$$

Here $m_1^{(0)}$ means a constant gaussian random variable and we assume

$$E[m_1(t)] = E[m_1^{(0)}] \tag{12}$$

then

$$E[m_1^{(1)}(t)] = 0 \tag{13}$$

Now we define state variables $\beta_1(t)$, $\beta_2(t)$ as follows.

$$\beta_1(t) = m_1^{(1)}(t) \tag{14}$$

$$\beta_2(t) = m_1^{(0)} \tag{15}$$

And $\beta_1(t)$ is assumed to be given by the following Gauss-Markov process.

$$\dot{\beta}_1(t) = -f_\beta\ \beta_1(t) + G_\beta\ u_1^{(\beta)}(t) \tag{16-1}$$

$f_\beta > 0$

$$\beta_1(t_0) = \beta_{1,0} \tag{16-2}$$

Furthermore

$$\dot{\beta}_2 = 0 \tag{17-1}$$

$$\beta_2(t_0) = \beta_{2,0} \tag{17-2}$$

Then the observed process is described as follows.

$$m_1(t) = \beta_1(t) + \beta_2(t) \tag{18}$$

The system input noise $u_1^{(\beta)}(t)$ is assumed to be a white gaussian noise with zero mean and the covariance $q^{(\beta)}$ which is a nonnegative constant and is defined by the following formula:

$$E[u_1^{(\beta)}(t)\ u_1^{(\beta)}(\tau)] = q^{(\beta)}\ \delta(t-\tau) \tag{19}$$

The initial value $\beta_{1,0}$ is a gaussian random variable with the mean $\overline{\beta_{1,0}} = E[\beta_{1,0}]$ and the nonnegative constant variance $P_0^{(\beta_1)}$ where $P_0^{(\beta_1)}$ is defined as follows

$$P_0^{(\beta_1)} = E[(\beta_{1,0} - \overline{\beta_{1,0}})^2] \tag{20-1}$$

Now we let $\beta_{2,0}$ be a gaussian random variable with the mean $\overline{\beta_{2,0}}=E[\beta_{2,0}]$ and the variance $P_0^{(\beta_2)}$ defined by

$$P_0^{(\beta_2)}=E[(\beta_{2,0}-\overline{\beta_{2,0}})^2] \tag{20-2}$$

Here $u_1^{(\beta)}(t)$ is statistically independent to $\beta_{1,0}$ and $\beta_{2,0}$. Furthermore $\beta_{1,0}$ and $\beta_{2,0}$ are independent each other.

(d) Additive noise

Additive noise is clearly independent to the signal and in practice its realization is mainly due to the thermal noise. We define an additive noise $n_1(t)$ by introducing the following Gauss-Markov process:

$$\dot{\theta}_1(t)=-f_\theta\ \theta_1(t)+G_\theta\ u_1^{(\theta)}(t) \tag{21-1}$$

$f_\theta>0$

$$\theta_1(t_0)=\theta_{1,0} \tag{21-2}$$

Thus

$$n_1(t)\lambda\ \theta_1(t)+v_1(t) \tag{22}$$

where λ is an arbitrary constant and $\theta_{1,0}$ is the initial value of the state variable $\theta_1(t)$, where $\theta_{1,0}$ is assumed to be a gaussian random variable with the mean $\overline{\theta_{1,0}}=E[\theta_{1,0}]$ and the nonnegative constant variance $P_0^{(\theta)}$ which is defined by

$$P_0^{(\theta)}=E[(\theta_{1,0}-\overline{\theta_{1,0}})^2] \tag{23}$$

Then $v_1(t)$ means modeling error. The system input noise $u_1^{(\theta)}$ is assumed to be a white gaussian noise with zero mean and the nonnegative constant covariance $q^{(\theta)}$ which is defined by

$$E[u_1^{(\theta)}(t)\ u_1^{(\theta)}(\tau)]=q^{(\theta)}\ \delta(t-\tau) \tag{24-1}$$

The observed noise $v_1(t)$ is also assumed to be a white gaussian noise with zero mean and the positive constant covariance r which is defined by the following.

$$E[v_1(t)\ v_1(\tau)]=r\,\delta(t-\tau) \tag{24-2}$$

And then $u_1^{(\theta)}(t)$ and $v_1(t)$ are statistically independent each other. Furthermore the initial value $\theta_{1,0}$ is independent to $u_1^{(\theta)}$ and $v_1(t)$.

(e) Received signal

Finally the received signal $y_1(t)$ can be described as follows.

$$\begin{aligned} y_1(t)&=z_1(t)+n_1(t)\\ &=w_1(t)\ m_1(t)+n_1(t)\\ &=\alpha_1(t)\ \beta_1(t)+\alpha_1(t)\ \beta_2(t)+\lambda\ \theta_1(t)+v_1(t) \end{aligned} \tag{25}$$

where

$$z_1(t)=w_1(t)\ m_1(t)$$
$$w_1(t)=\alpha_1(t)$$
$$m_1(t)=\beta_1(t)+\beta_2(t)$$

From the set of dynamical systems (2-1), (8-1), (16-1), (17-1), and (21-1), we can construct the following nonlinear dynamical system.

$$\dot{x}(t)=f\,[t,\ x(t)]+gu(t) \tag{26}$$

where new state vector $x(t)\ \epsilon\ R^5$ is

$$\begin{aligned} x^T(t)&=(x_1(t),\ x_2(t),\ x_3(t),\ x_4(t),\ x_5(t))\\ &=(\xi_1(t),\ \alpha_1(t),\ \beta_1(t),\ \beta_2(t),\ \theta_1(t)) \end{aligned} \tag{27}$$

In the equation (27) the superscript T means the vector (matrix) transposed. Here

$$f[t,\ x(t)]=\begin{bmatrix} -Ax_1(t)\\ -k_2x_2(t)+k_1\ \mathrm{Csin}\ [\omega_0\,t+dx_1(t)]\\ -fx_3(t)\\ 0\\ -fx_5(t) \end{bmatrix} \tag{28}$$

$$g=\begin{bmatrix} G_\xi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & G_\beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_\theta \end{bmatrix} \tag{29}$$

$$u(t)=\begin{bmatrix} u_1^{(\xi)}(t) \\ u_1^{(\beta)}(t) \\ u_1^{(\theta)}(t) \end{bmatrix} \tag{30}$$

The initial condition is as follows.

$$\begin{aligned} x^T(t_0) &= (\xi_1(t_0),\ \alpha_1(t_0),\ \beta_1(t_0),\ \beta_2(t_0),\ \theta_1(t_0)) \\ &= (x_{1,0},\ x_{2,0},\ x_{3,0},\ x_{4,0},\ x_{5,0}) \end{aligned} \tag{31}$$

and $x(t_0)$ is clearly gaussian random vector with the mean

$\overline{x_0}=(\overline{x_{1,0}},\ \overline{x_{2,0}},\ \overline{x_{3,0}},\ \overline{x_{4,0}},\ \overline{x_{5,0}})=(\overline{\xi_{1,0}},\ \overline{\alpha_{1,0}},\ \overline{\beta_{1,0}},\ \overline{\beta_{2,0}},\ \overline{\theta_{1,0}})$

and 5×5 symmetric nonnegative definite covariance matrix

$P_0=(P_{0,ij})\ i,\ j=1\cdots5$, where $P_{0,11}=P_0^{(\xi)}$, $P_{0,22}=P_0^{(\alpha)}$, $P_{0,33}=P^{(\beta_1)}$, $P_{0,44}=P_0^{(\beta_2)}=0$, $P_{0,55}=P_0^{(\theta)}$

Moreover $u(t)\ \epsilon\ R^3$ is a white gaussian noise with zero mean and a 3×3 constant covariance matrix Q which is non-negative definite and defined as follows.

$$E[u(t)\ u^T(\tau)]=Q\,\delta(t-\tau) \tag{32}$$

$$Q=\begin{bmatrix} q^{(\xi)} & 0 & 0 \\ 0 & q^{(\beta)} & 0 \\ 0 & 0 & q^{(\theta)} \end{bmatrix} \tag{33}$$

The functional $f[t,\ x(t)]$ is a 5×1 time continuous vector valued functional ang g is a constant 5×3 matrix. The initial value $x(t_0)\ \epsilon\ R^5$ is a gaussian constant vector and statistically independent to $u(t)$. And any two of $u_1^{(\xi)}$, $u_1^{(\beta)}$ and $u_1^{(\theta)}$ are assumed to be independent to each other.

The observed process can be written as

$$y_1(t)=h_1[t,\ x(t)]+v_1(t) \tag{34}$$

$$h_1[t,\ x(t)]=x_2(t)\ x_3(t)+x_2(t)\ x_4(t)+x_5(t) \tag{35}$$

where $h_1[t,\ x(t)]$ is time continuous functional, $v_1(t)$ means the observed noise and its properties have already been defined. We must here note that the dynamical system (26) and the observed process (34) are, as well known, rather formal and mathematical fictions. Then equation (26) and (34) should be formulated by Itô stochastic differential equation[10]:

$$dx(t)=f[t,\ x(t)]\ dt+g\,dB^{(u)}(t) \tag{36}$$

$$dy_1^{(r)}(t)=h_1[t,\ x(t)]\ dt+dB_1^{(v)}(t) \tag{37}$$

where $B^{(u)}(t)$ and $B_1^{(v)}(t)$ are Brownian motion processes and the following relations are provided:

$$y_1(t)=\frac{dy_1^{(r)}(t)}{dt} \tag{38}$$

$$u(t)=\frac{dB^{(u)}(t)}{dt} \tag{39}$$

$$v_1(t)=\frac{dB_1^{(v)}(t)}{dt} \tag{40}$$

$$E[dB^{(u)}(t)\ dB^{(u)}(t)^T]=Q\,dt \tag{41}$$

$$E[dB_1^{(v)})^2]=r\,dt \tag{42}$$

[4] Filter algorithm

Now we reform the dynamical system (36) and the observed processes (37) via general vector formulas.

$$dx(t)=[t,\ x(t)]\ dt+g(t)\ dB^{(u)}(t) \tag{43}$$

$$dy^{(r)}(t)=h[t,\ x(t)]\ dt+dB^{(v)}(t) \tag{44}$$

where $x(t)\ \epsilon\ R^{n_x}$, $B^{(u)}(t)\ \epsilon\ R^{m_u}$, $y^{(r)}(t)\ \epsilon\ R^{n_y}$, $B^{(v)}(t)\ \epsilon\ R^{n_y}$

where n_x, n_y and m_u are positive integers and $f[t,\ x(t)]$ and $f[t,\ x(t)]$ are time continuous functionals with a fitted size, also $g(t)$ is a time continuous matrix with a fitted size.

Now let

$$Y_\tau=\{y^{(r)}(s),\ \ t_0<s<\tau\} \tag{45}$$

and let the conditional expectation as

$$x(t)=E[x(t)\ |\ Y_t] \tag{46}$$

Then we know already the following nonlinear filter process[11]

$$dx=\widehat{f}+(\widehat{xh^T}-\widehat{x}\widehat{h}^T)\ R^{-1}(t)(dy^{(r)}-\widehat{h}dt) \tag{47}$$

$$\begin{aligned}(dp)_{ij}=&[(\widehat{x_if_j}-\widehat{x}_i\widehat{f}_j)+(\widehat{f_ix_j}-\widehat{f}_i\widehat{x}_j)\\&+(\widehat{GQG^T})_{ij}-(\widehat{x_ih}-\widehat{x}_j\widehat{h})\ R^{-1}(t)(\widehat{hx_j}-\widehat{h}\widehat{x}_j)]\ dt\\&+(\widehat{x_ix_jh}-\widehat{x}_i\widehat{x_jh}-\widehat{x}_i\widehat{x}_j\widehat{h}-\widehat{x}_j\widehat{x_ih}+2\widehat{x}_i\widehat{x}_j\widehat{h})^T\ R^{-1}(t)(dy^{(r)}-\widehat{h}dt)\end{aligned} \tag{48}$$

where symbol $\wedge$ means the conditional expectation and the indices i, j or ij are i th, j th component of vectors and ij element of matrix respectively. $P(t)$ is the estimate error covariance matrix which is defined as

$$P(t)=E[x(t)-\widehat{x}(t))(x(t)-\widehat{x}(t))^T\ |\ Y_t] \tag{49}$$

Note that the time continuous $n_y\times n_y$ positive covariance matrix $R(t)$ is clearly the extension of the constant variance r to the matrix formula. Then the initial condition is

$$\widehat{x}(t_0)=\overline{x_0} \tag{50}$$

$$P(t_0)=P_0 \tag{51}$$

where

$$\overline{x_0}=E[x(t_0)] \tag{52}$$

P_0 is a symmetric constant matrix and non-negative.

[5] Simulation

We now must note that the nonlinear filter obtained in the previous chapter can not be realized as it is, because the filter requires the calculation of infinite moments. Then an approximate filter is required. Thus we adopt here the first-order appoximate filter. We expand each component of functional f and h about the estimate $\widehat{x}$ of x in Tailor series and taking the conditional expectation. Then we have

$$\widehat{f}_i[t,\ x]=f_i[t,\ \widehat{x}] \tag{53}$$

$$\widehat{h}_i[t,\ x]=h_i[t,\ \widehat{x}] \tag{54}$$

where we omitted higher order terms provided that the estimate error $x-\widehat{x}$ is small. Hence we obtain easily the following relations.

$$\widehat{x_if_j}-\widehat{x}_i\widehat{f}_j=\sum_{k=1}^{n_x}P_{ik}\frac{\partial f_i[t,\ \widehat{x}]}{\partial x_k} \tag{55}$$

$$\widehat{x_ih_j}-\widehat{x}_i\widehat{h}_j=\sum_{k=1}^{n_y}P_{ik}\frac{\partial h_j[t,\ \widehat{x}]}{\partial x_k} \tag{56}$$

$$\widehat{x_ix_jh_k}-\widehat{x}_i\widehat{x_jh_k}-\widehat{x}_i\widehat{x}_j\widehat{h}_k-\widehat{x}_j\widehat{x_ih_k}+2\widehat{x}_i\widehat{x}_j\widehat{h}_k=0 \tag{57}$$

Finally we have

$$d\widehat{x}(t)=f[t,\ \widehat{x}(t)]\ dt+P(t)\ h_x{}^T[t,\ \widehat{x}(t)]\,R^{-1}(t)\,[dy^{(r)}(t)-h[t,\ \widehat{x}(t)]\,dt] \tag{58}$$

$$\begin{aligned}dP(t)=&f_x[t,\ \widehat{x}(t)]\,P(t)\,dt+P(t)\ f_x{}^T[t,\ \widehat{x}(t)]\,dt\\&-P(t)\,h_x{}^T[t,\ \widehat{x}(t)]\,R^{-1}(t)\,h_x[t,\ \widehat{x}(t)]\ P(t)\,dt+g(t)\,Q(t)\,g^T(t)\,dt\end{aligned} \tag{59}$$

$$\widehat{x}(t)=\overline{x_0} \tag{60}$$

$$P(t_0)=P_0 \tag{61}$$

where f_x, h_x means the partial derivative via x. Then the filter to the dynamical system (36) with the observed process (37) reduced to the following filter system.

$$d\widehat{x}(t)=f[t,\ \widehat{x}(t)]\,dt+P(t)\,h^T{}_{1,x}[t,\ \widehat{x}(t)]\,r^{-1}[dy_1{}^{(r)}-h_1[t,\ \widehat{x}(t)]\,dt] \tag{62}$$

$$\begin{aligned} dP(t)=&f_x[t,\ \widehat{x}(t)]\,P(t)\,dt+P(t)\,f_x{}^T[t.\ \widehat{x}(t)]\,dt \\ &-P(t)\,h_{1,x}{}^T[t,\ \widehat{x}(t)]\,r^{-1}h_{1,x}[t,\ \widehat{x}(t)]\,P(t)+gQg^T \end{aligned} \tag{63}$$

$$\widehat{x}(t_0)=\overline{x_0} \tag{64}$$

$$P(t_0)=P_0 \tag{65}$$

where

$$\widehat{x}(t)^T=(\widehat{x}_1(t),\ \widehat{x}_2(t),\ \widehat{x}_3(t),\ \widehat{x}_4(t),\ \widehat{x}_5(t)) \tag{66}$$

$$f_x[t,\ \widehat{x}(t)]=\begin{bmatrix} -A & 0 & 0 & 0 & 0 \\ \Gamma(t) & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -f_\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -f_\theta \end{bmatrix} \tag{67-1}$$

$$\Gamma(t)=k_1\ \mathrm{Cd}\cos[\omega_0\ t+d\widehat{x}_1(t)] \tag{67-2}$$

$$\mathrm{h}_1[t,\ \widehat{x}(t)]=\widehat{x}_2(t)\,\widehat{x}_3(t)+\widehat{x}_2(t)\,\widehat{x}_4(t)+\lambda\,\widehat{x}_5(t) \tag{68}$$

$$h_{1,x}[t,\ \widehat{x}(t)]=(0,\ \widehat{x}_3(t)+\widehat{x}_4(t),\ \widehat{x}_2(t),\ \widehat{x}_2(t),\ \lambda) \tag{69}$$

$$P(t)=(P_{ij}(t)),\ i,\ j=1,2\cdots\cdots5 \tag{70}$$

$$P_{ij}(t)=P_{ji}(t)$$

$$\overline{x_0}^T=(\overline{x_{0,1}},\ \overline{x_{0,2}},\ \overline{x_{0,3}},\ \overline{x_{0,4}},\ \overline{x_{0,5}}) \tag{71}$$

$$P_0=(P_{0,ij})\quad i,\ j=1,2\cdots\cdots5 \tag{72}$$

$$P_{0,ij}=P_{0,ji}$$

and g, Q are already defined by the equation[29],[33].

Now we represent the simulation of filters where constant values are assumed as follows.

$A=40,\ C=1,\ d=0.1,\ k_1=k_2=1,\ f_\beta=f_\theta=100,\ G_\xi=0.3,\ G_\beta=G_\theta=0.5,\ f=100,\ \lambda=2.0,\ q^{(\xi)}=q^{(\beta)}=q^{(\theta)}=1,\ r=0.1$

Initial conditions are provided by the following constants.

$x_{0,1}=x_{0,2}=1,\ x_{0,3}=1.5,\ x_{0,4}=3.829,\ x_{0,5}=0.1$

$\overline{x_{0,1}}=\overline{x_{0,2}}=\overline{x_{0,3}}=\overline{x_{0,5}}=0,\ \overline{x_{0,4}}=2,$

$P_{0,ij}=0(i=j,\ i,\ j=1.2\cdots\cdots5),\ P_{0,11}=P_{0,22}=P_{0,33}=P_{0,55}=1.2,\ P_{0,44}=1.5.$

And another numerical experiment is given when $\overline{x_{0,1}}=1.2$ but the other initial values and constants are the same. Then we show the simulation of the filter to estimate the random signal message by digital computer. The approximation made in numerical calculation is snch that dt $\approx$ 0.001, $dx(t)\approx x(t_{j+1})-x(t_j)$. $dx(t)$ and $dP(t)$ are also approximated in the same manner. And we adopt 201 numerical data : $j=1,2\cdots\cdots201$ where t_0 corresponds to the step 0. The results of numerical experiments are shown in **Fig. 2** and **Fig. 3.**

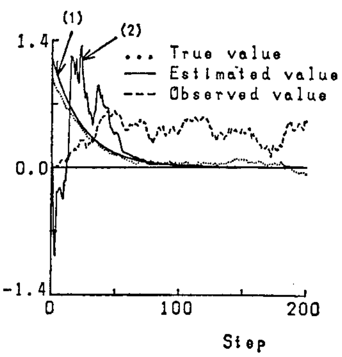

Fig. 2 True value $x_1(t)$, observed value $y_1{}^{(r)}(t)$ and estimate value $x_1(t)$: (1), (2)($\times 10^{-4}$)

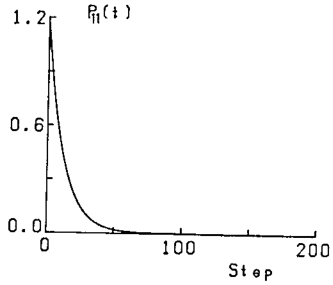

Fig. 3 Estimate error variance $P_{11}(t)$

The received signal (Observed value) and the signal message (True value) are shown in Fig. 2. Two kinds of the behavior, denoted by (1) and (2) of the estimated value $\hat{x}_1(t)$ $(=\hat{\xi}_1(t))$ are represented: (1) denotes the estimate to the case of $\overline{x_{0,1}}=1.2$. (2) denotes the estimate to the case of $\overline{x_{0,1}}=0$. But the amplitude of (2) is very small (the ratio of the estimated values (2) to (1) is 10^{-4}). Obviously it is recognized that the estimated values (1) and also (2) pursuit well the true value and the effects of the initial value $\overline{x_{0,1}}$ have important role in the estimate of signal message. In Fig. 3, the variance $P_{11}(t)$ of the estimated error $x_1(t)-\hat{x}_1(t)$ is showed to the case of $\overline{x_{0,1}}=1.2$ and $\overline{x_{0,1}}=0$ but we can not see the difference between the two cases. Clearly $P_{11}(t)$ converges to zero rapidly.

[6] Conclusion

We treated here the filtering or estimation by applying the state estimate method to estimate of phase modulated random signal message in a transmission system with multiplicative and additive noises. Then it was understood that it is easy to obtain the numerical solution of the filter. The numerical results given here are very interesting because the solutions are given in real time.

In this paper only continuous systems are discussed. Since the continuity is not essential in the present theory, discrete systems can also be treated in the same manner·

Finally we have to express a few problems which might arise. First the dimensions of the dynamical systems being related to a general transmission systems may increase when the system becomes large. To find how to avoid this situation is very important problem. Secondly it has to be noted that the errors of the numerical calculation by computer may increase depending on the properties of the random signal message, and because of this the diagonal elements of the estimate error covariance may become negative. The answer to theses questions should be cleared in future.

[7] References

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